Grain Size

How Large is a Grain?

In the 1950s, two metallurgists working independently^{*} found a relationship between grain size and strength in metals: $\sigma_{YS} = \sigma_o + \frac{K}{\sqrt{d}}$ where σ_o is the stress required to start dislocation movement (the intercept of the line on the graph), *d* is the average grain diameter, and *K* is the strengthening coefficient (the slope of the line). Both σ_o and *K* are materials properties, so each material has a different line on the graph. If we know σ_o and *K*, we can predict the yield strength of a metal based on its average grain diameter; alternately, if we know what yield strength is needed for a particular application, we can determine the required grain diameter.

Since grains are three-dimensional objects in space, we see a distorted view of the actual grains when we section a piece of metal. Imagine you have a jar of ½ inch diameter glass marbles. You pour epoxy into the jar so it fills all of the spaces between the marbles, forming a single solid chunk of material. After the epoxy has cured, slice the jar with a laser. Sometimes, the laser cuts through the equator of a marble; sometimes it kisses a pole; but most of the time it cuts somewhere between the equator and pole. When you take the cross-section of a sphere, the twodimensional shape is a circle...so we get a bunch of circles of varying diameters. If you calculate the average diameter of the circles, you will get some number that is smaller than the actual diameter of the marbles.

The same effect occurs with grains in metals, except grains are not spherical and they are not all the same size. Therefore, any measurement of grain size using a crosssection must inevitably be a measurement of **observed** grain size, not the actual size of three-dimensional grains.





^{*} E.O. Hall at the University of Sheffield studied yielding behavior of mild steels (ductile failure). N.J. Petch of the University of Leeds studied low-temperature brittle failure of steels. Both metallurgists are recognized as developing the Hall-Petch equation.

ASTM Standard E 112 specifies how the grain size of metals is determined. If you look through a microscope at a polished and etched metal sample at $100 \times$ magnification, the number of grains you see within a square inch is uppercase N in the following equation:

 $N = 2^{n-1}$

Lower-case *n* is the ASTM grain size.

This equation is not particularly convenient...the known value is N, and the unknown is n. To solve for grain size, take the log of each side:

 $\log N = \log 2^{n-1}$

With the log of an exponent, you can take the exponent and put it in front of the log:

 $\log N = (n-1)\log 2$

Now divide both sides by log 2:

$$\frac{\log N}{\log 2} = n - 1$$

Finally, solve for *n*:

 $n = 1 + \frac{\log N}{\log 2}$

